How do you measure a photo finish?

Sprint times are often measured to the nearest hundredth of a second (0.01 s). Chemistry also requires making accurate and often very small measurements.
3.1 Using and Expressing Measurements > Scientific Notation

Scientific Notation

How do you write numbers in scientific notation?

3.1 Using and Expressing Measurements > Scientific Notation

• A measurement is a quantity that has both a number and a unit.

  • Your height (66 inches), your age (15 years), and your body temperature (37°C) are examples of measurements.
In chemistry, you will often encounter very large or very small numbers.

A single gram of hydrogen, for example, contains approximately 602,000,000,000,000,000,000 hydrogen atoms.

You can work more easily with very large or very small numbers by writing them in scientific notation.

In **scientific notation**, a given number is written as the product of two numbers: a coefficient and 10 raised to a power.

For example, the number 602,000,000,000,000,000,000 can be written in scientific notation as 6.02 x 10^{23}.

The coefficient in this number is 6.02. The power of 10, or exponent, is 23.
In scientific notation, the coefficient is always a number greater than or equal to one and less than ten. The exponent is an integer.

- A positive exponent indicates how many times the coefficient must be multiplied by 10.

- A negative exponent indicates how many times the coefficient must be divided by 10.

When writing numbers greater than ten in scientific notation, the exponent is positive and equals the number of places that the original decimal point has been moved to the left.

\[ 6,300,000 = 6.3 \times 10^6 \]

\[ 94,700 = 9.47 \times 10^4 \]
3.1 Using and Expressing Measurements > Scientific Notation

Numbers less than one have a negative exponent when written in scientific notation. The value of the exponent equals the number of places the decimal has been moved to the right.

\[ 0.000\,008 = 8 \times 10^{-6} \]

\[ 0.00736 = 7.36 \times 10^{-3} \]

Multiplication and Division

To multiply numbers written in scientific notation, multiply the coefficients and add the exponents.

\[(3 \times 10^4) \times (2 \times 10^2) = (3 \times 2) \times 10^{4+2} = 6 \times 10^6\]

\[(2.1 \times 10^3) \times (4.0 \times 10^{-7}) = (2.1 \times 4.0) \times 10^{3+(-7)} = 8.4 \times 10^{-4} \]
To divide numbers written in scientific notation, divide the coefficients and subtract the exponent in the denominator from the exponent in the numerator.

$$\frac{3.0 \times 10^5}{6.0 \times 10^2} = \left( \frac{3.0}{6.0} \right) \times 10^{5-2} = 0.5 \times 10^3 = 5.0 \times 10^2$$

Addition and Subtraction

- If you want to add or subtract numbers expressed in scientific notation and you are not using a calculator, then the exponents must be the same.

- In other words, the decimal points must be aligned before you add or subtract the numbers.
**Addition and Subtraction**

For example, when adding $5.4 \times 10^3$ and $8.0 \times 10^2$, first rewrite the second number so that the exponent is a 3. Then add the numbers.

$$(5.4 \times 10^3) + (0.80 \times 10^3) = (5.4 + 0.80) \times 10^3 = 6.2 \times 10^3$$

**Sample Problem 3.1**

**Using Scientific Notation**

Solve each problem and express the answer in scientific notation.

**a.** $(8.0 \times 10^{-2}) \times (7.0 \times 10^{-5})$

**b.** $(7.1 \times 10^{-2}) + (5 \times 10^{-3})$
3.1 Using and Expressing Measurements > Sample Problem 3.1

1 Analyze  Identify the relevant concepts.

To multiply numbers in scientific notation, multiply the coefficients and add the exponents. To add numbers in scientific notation, the exponents must match. If they do not, then adjust the notation of one of the numbers.

2 Solve  Apply the concepts to this problem.

Multiply the coefficients and add the exponents.

a. \((8.0 \times 10^{-2}) \times (7.0 \times 10^{-5}) = (8.0 \times 7.0) \times 10^{-2 + (-5)} = 56 \times 10^{-7} = 5.6 \times 10^{-6}\)
2 Solve  Apply the concepts to this problem.

Rewrite one of the numbers so that the exponents match. Then add the coefficients.

b. $\left(7.1 \times 10^{-2}\right) + \left(5 \times 10^{-3}\right) = \left(7.1 \times 10^{-2}\right) + \left(0.5 \times 10^{-2}\right)$

$= (7.1 + 0.5) \times 10^{-2}$

$= 7.6 \times 10^{-2}$

The mass of one molecule of water written in scientific notation is $2.99 \times 10^{-23}$ g. What is the mass in standard notation?
The mass of one molecule of water written in scientific notation is $2.99 \times 10^{-23}$ g. What is the mass in standard notation?

The mass of one molecule of water in standard notation is 0.000 000 000 000 000 000 000 0299 gram.

Accuracy, Precision, and Error

How do you evaluate accuracy and precision?
In chemistry, the meanings of accuracy and precision are quite different.

- **Accuracy** is a measure of how close a measurement comes to the actual or true value of whatever is measured.

- **Precision** is a measure of how close a series of measurements are to one another, irrespective of the actual value.

To evaluate the accuracy of a measurement, the measured value must be compared to the correct value. To evaluate the precision of a measurement, you must compare the values of two or more repeated measurements.
Accuracy and Precision

Darts on a dartboard illustrate the difference between accuracy and precision.

- Good Accuracy, Good Precision
- Poor Accuracy, Good Precision
- Poor Accuracy, Poor Precision

The closeness of a dart to the bull’s-eye corresponds to the degree of accuracy. The closeness of several darts to one another corresponds to the degree of precision.

Determining Error

- Suppose you use a thermometer to measure the boiling point of pure water at standard pressure.
  - The thermometer reads 99.1°C.
  - You probably know that the true or accepted value of the boiling point of pure water at these conditions is actually 100.0°C.
There is a difference between the **accepted value**, which is the correct value for the measurement based on reliable references, and the **experimental value**, the value measured in the lab.

The difference between the experimental value and the accepted value is called the **error**.

Error = experimental value – accepted value

Error can be positive or negative, depending on whether the experimental value is greater than or less than the accepted value.

For the boiling-point measurement, the error is 99.1°C – 100°C, or −0.9°C.
Determining Error

- Often, it is useful to calculate the relative error, or percent error, in addition to the magnitude of the error.

- The **percent error** of a measurement is the absolute value of the error divided by the accepted value, multiplied by 100%.

\[
\text{Percent error} = \frac{|\text{error}|}{\text{accepted value}} \times 100\%
\]

Sample Problem 3.2
Calculating Percent Error

The boiling point of pure water is measured to be 99.1°C. Calculate the percent error.

Think about it: Using the absolute value of the error means that percent error will always be a positive value.
Sample Problem 3.2

1. **Analyze** List the knowns and unknown.

   The accepted value for the boiling point of pure water is 100°C. Use the equations for error and percent error to solve the problem.

   **KNOWNS**
   - Experimental value = 99.1°C
   - Accepted value = 100.0°C

   **UNKNOWN**
   - Percent error = ?

2. **Calculate** Solve for the unknown.

   Start with the equation for percent error.

   \[
   \text{Percent error} = \frac{|\text{error}|}{\text{accepted value}} \times 100\%
   \]
3.1 Using and Expressing Measurements > Sample Problem 3.2

2 Calculate Solve for the unknown.

Substitute the equation for error, and then plug in the known values.

\[
\text{Percent error} = \frac{|\text{experimental value} - \text{accepted value}|}{\text{accepted value}} \times 100\%
\]

\[
= \frac{|99.1^\circ C - 100.0^\circ C|}{100.0^\circ C} \times 100\%
\]

\[
= \frac{0.9^\circ C}{100.0^\circ C} \times 100\% = 0.9\%
\]

3 Evaluate Does the result make sense?

The experimental value was off by about 1°C, or 1/100 of the accepted value (100°C). The answer makes sense.
Are precise measurements always accurate?

No, measurements are precise if they are easily reproducible, but not accurate if they do not reflect the accepted value.
Significant Figures

Why must measurements be reported to the correct number of significant figures?

• You can easily read the temperature on this thermometer to the nearest degree.

• You can also estimate the temperature to about the nearest tenth of a degree by noting the closeness of the liquid inside to the calibrations.

• Suppose you estimate that the temperature lies between 22°C and 23°C, at 22.9°C.
This estimated number, 22.9°C, has three digits.

The first two digits (2 and 2) are known with certainty, while the rightmost digit (9) has been estimated and involves some uncertainty.

These reported digits all convey useful information, however, and are called significant figures.

The significant figures in a measurement include all of the digits that are known, plus a last digit that is estimated.

Measurements must always be reported to the correct number of significant figures because calculated answers often depend on the number of significant figures in the values used in the calculation.
3.1 Using and Expressing Measurements > Significant Figures

- Instruments differ in the number of significant figures that can be obtained from their use and thus in the precision of measurements.

- A meterstick calibrated in 0.1-m intervals is more precise than one calibrated in a 1-m interval.

Determining Significant Figures in Measurements

To determine whether a digit in a measured value is significant, you need to apply the following rules.

1. Every nonzero digit in a reported measurement is assumed to be significant.

   Each of these measurements has three significant figures:

   - 24.7 meters
   - 0.743 meter
   - 714 meters
2. Zeros appearing between nonzero digits are significant.

Each of these measurements has four significant figures:

7003 meters
40.79 meters
1.503 meters

3. Leftmost zeros appearing in front of nonzero digits are not significant. They act as placeholders. By writing the measurements in scientific notation, you can eliminate such placeholder zeros.

Each of these measurements has only two significant figures:

0.0071 meter = 7.1 x 10^{-3} meter
0.42 meter = 4.2 x 10^{-1} meter
0.000099 meter = 9.9 x 10^{-5} meter
4. Zeros at the end of a number and to the right of a decimal point are always significant.

Each of these measurements has four significant figures:

- 43.00 meters
- 1.010 meters
- 9.000 meters

5. Zeros at the rightmost end of a measurement that lie to the left of an understood decimal point are not significant if they serve as placeholders to show the magnitude of the number.

The zeros in these measurements are not significant:

- 300 meters (one significant figure)
- 7000 meters (one significant figure)
- 27,210 meters (four significant figures)
Determining Significant Figures in Measurements

5 (continued). If such zeros were known measured values, then they would be significant. Writing the value in scientific notation makes it clear that these zeros are significant.

The zeros in this measurement are significant.

300 meters = 3.00 \times 10^2 \text{ meters}

(three significant figures)

6. There are two situations in which numbers have an unlimited number of significant figures. The first involves counting. A number that is counted is exact.

This measurement is a counted value, so it has an unlimited number of significant figures.

23 people in your classroom
6 (continued). The second situation involves exactly defined quantities such as those found within a system of measurement.

Each of these numbers has an unlimited number of significant figures.

\[
\begin{align*}
60 \text{ min} &= 1 \text{ hr} \\
100 \text{ cm} &= 1 \text{ m}
\end{align*}
\]

Suppose that the winner of a 100-meter dash finishes the race in 9.98 seconds. The runner in second place has a time of 10.05 seconds. How many significant figures are in each measurement? Is one measurement more accurate than the other? Explain your answer.
Suppose that the winner of a 100-meter dash finishes the race in 9.98 seconds. The runner in second place has a time of 10.05 seconds. How many significant figures are in each measurement? Is one measurement more accurate than the other? Explain your answer.

There are three significant figures in 9.98 and four in 10.05. Both measurements are equally accurate because both measure the actual time of the runner to the hundredth of a second.

Counting Significant Figures in Measurements

How many significant figures are in each measurement?

a. 123 m
b. 40,506 mm
c. 9.8000 \times 10^4 m
d. 22 metersticks
e. 0.07080 m
f. 98,000 m

Make sure you understand the rules for counting significant figures (in the previous slides) before you begin, okay?
Analyze  Identify the relevant concepts.

The location of each zero in the measurement and the location of the decimal point determine which of the rules apply for determining significant figures. These locations are known by inspecting each measurement value.

Solve  Apply the concepts to this problem.

Apply the rules for determining significant figures. All nonzero digits are significant (rule 1). Use rules 2 through 6 to determine if the zeros are significant.

a. 123 m has three significant figures (rule 1)
b. 40,506 m has five (rule 2)
c. 9.8000 x 10^4 m has five (rule 4)
d. 22 metersticks has unlimited (rule 6)
e. 0.07080 m has four (rules 2, 3, 4)
f. 98,000 m has two (rule 5)
3.1 Using and Expressing Measurements > Significant Figures

Significant Figures in Calculations

• In general, a calculated answer cannot be more precise than the least precise measurement from which it was calculated.

• The calculated value must be rounded to make it consistent with the measurements from which it was calculated.

Rounding

• To round a number, you must first decide how many significant figures the answer should have.

• This decision depends on the given measurements and on the mathematical process used to arrive at the answer.

• Once you know the number of significant figures your answer should have, round to that many digits, counting from the left.
**Rounding**

- If the digit immediately to the right of the last significant digit is less than 5, it is simply dropped and the value of the last significant digit stays the same.

- If the digit in question is 5 or greater, the value of the digit in the last significant place is increased by 1.

**Sample Problem 3.4**

**Rounding Measurements**

Round off each measurement to the number of significant figures shown in parentheses. Write the answers in scientific notation.

a. 314.721 meters (four)

b. 0.001775 meter (two)

c. 8792 meters (two)
1. **Analyze** Identify the relevant concepts.

Using the rules for determining significant figures, round the number in each measurement. Then apply the rules for expressing numbers in scientific notation.

2. **Solve** Apply the concepts to this problem.

Starting from the left, count the first four digits that are significant. The arrow points to the digit immediately following the last significant digit.

a. 314.721 meters

   2 is less than 5, so you do not round up.

   314.7 meters = 3.147 x 10^2 meters
2 Solve Apply the concepts to this problem.

Starting from the left, count the first two digits that are significant. The arrow points to the digit immediately following the second significant digit.

b. 0.001 775 meters
   \[ \text{↑} \]
   7 is greater than 5, so round up.
   
   0.0018 meter = 1.8 \times 10^{-3} \text{ meter}

c. 8792 meters
   \[ \text{↑} \]
   9 is greater than 5, so round up.
   
   8800 meters = 8.8 \times 10^{3} \text{ meters}
Significant Figures in Calculations

Addition and Subtraction

The answer to an addition or subtraction calculation should be rounded to the same number of decimal places (not digits) as the measurement with the least number of decimal places.

Sample Problem 3.5

Significant Figures in Addition and Subtraction

Perform the following addition and subtraction operations. Give each answer to the correct number of significant figures.

a. 12.52 meters + 349.0 meters + 8.24 meters
b. 74.626 meters – 28.34 meters
1. **Analyze** Identify the relevant concepts.
   Perform the specified math operation, and then round the answer to match the measurement with the least number of decimal places.

2. **Solve** Apply the concepts to this problem.
   Align the decimal points and add the numbers.

   a.  
   \[
   \begin{align*}
   12.52 \text{ meters} \\
   + 349.0 \text{ meters} \\
   + 8.24 \text{ meters} \\
   \hline
   369.76 \text{ meters}
   \end{align*}
   \]
2 Solve Apply the concepts to this problem.

The second measurement (349.0 meters) has the least number of digits (one) to the right of the decimal point. So, the answer must be rounded to one digit after the decimal point.

a. \[ 12.52 \text{ meters} + 349.0 \text{ meters} + 8.24 \text{ meters} + 369.76 \text{ meters} \]
   \[ = 369.8 \text{ meters} = 3.698 \times 10^2 \]

2 Solve Apply the concepts to this problem.

Align the decimal points and subtract the numbers.

b. \[ 74.636 \text{ meters} - 28.34 \text{ meters} - 46.286 \text{ meters} \]
3.1 Using and Expressing Measurements > Sample Problem 3.5

2 Solve Apply the concepts to this problem.

The second measurement (28.34 meters) has the least number of digits (two) to the right of the decimal point. So, the answer must be rounded to two digits after the decimal point.

b. \[ \begin{array}{c}
74.636 \text{ meters} \\
- 28.34 \text{ meters} \\
\hline
46.286 \text{ meters}
\end{array} \]

= 46.29 meters

= \(4.629 \times 10^1\) meters

---

3.1 Using and Expressing Measurements > Significant Figures

Significant Figures in Calculations

**Multiplication and Division**

- In calculations involving multiplication and division, you need to round the answer to the same number of significant figures as the measurement with the least number of significant figures.

- The position of the decimal point has nothing to do with the rounding process when multiplying and dividing measurements.
Sample Problem 3.6

Significant Figures in Multiplication and Division

Perform the following operations. Give the answers to the correct number of significant figures.

a. 7.55 meters x 0.34 meter
b. 2.10 meters x 0.70 meter
c. 2.4526 meters² ÷ 8.4 meters
d. 0.365 meter² ÷ 0.0200 meter

1 Analyze Identify the relevant concepts.

Perform the specified math operation, and then round the answer to match the measurement with the least number of significant figures.
Sample Problem 3.6

2 Solve Apply the concepts to this problem.

a. 7.55 meters x 0.34 meter

The second measurement (0.34 meter) has the least number of significant figures (two). So, the answer must be rounded to two significant figures.

\[
a. 7.55 \text{ meters} \times 0.34 \text{ meter} = 2.567 \text{ meters}^2 = 2.6 \text{ meters}^2
\]

b. 2.10 meters x 0.70 meter

The second measurement (0.70 meter) has the least number of significant figures (two). So, the answer must be rounded to two significant figures.

\[
b. 2.10 \text{ meters} \times 0.70 \text{ meter} = 1.47 \text{ meters}^2 = 1.5 \text{ meters}^2
\]
3.1 Using and Expressing Measurements

Sample Problem 3.6

2 Solve Apply the concepts to this problem.

c. \(2.4526 \text{ meters}^2 \div 8.4 \text{ meters}\)

The second measurement (8.4 meters\(^2\)) has the least number of significant figures (two). So, the answer must be rounded to two significant figures.

\[2.4526 \text{ meters}^2 \div 8.4 \text{ meters} = 0.291076 \text{ meter} = 0.29 \text{ meter}\]

Sample Problem 3.6

2 Solve Apply the concepts to this problem.

d. \(0.365 \text{ meters}^2 \div 0.0200 \text{ meter}\)

Both measurements have three significant figures. So, the answer must be rounded to three significant figures.

\[0.365 \text{ meters}^2 \div 0.0200 \text{ meter} = 18.25 \text{ meters} = 18.3 \text{ meters}\]
In what case are zeros not significant in a measured value?

Sometimes zeros are not significant when they serve as placeholders to show the magnitude of the measurement.
In scientific notation, the coefficient is always a number greater than or equal to one and less than ten. The exponent is an integer.

To evaluate accuracy, the measured value must be compared to the correct value. To evaluate the precision of a measurement, you must compare the values of two or more repeated measurements.

Measurements must always be reported to the correct number of significant figures because calculated answers often depend on the number of significant figures in the values used in the calculation.
3.1 Using and Expressing Measurements > Key Equations

Error = experimental value – accepted value

Percent error = \( \frac{|\text{error}|}{\text{accepted value}} \times 100\% \)

3.1 Using and Expressing Measurements > Glossary Terms

- **measurement**: a quantitative description that includes both a number and a unit

- **scientific notation**: an expression of numbers in the form \( m \times 10^n \), where \( m \) is equal to or greater than 1 and less than 10, and \( n \) is an integer

- **accuracy**: the closeness of a measurement to the true value of what is being measured

- **precision**: describes the closeness, or reproducibility, of a set of measurements taken under the same conditions
3.1 Using and Expressing Measurements > Glossary Terms

- **accepted value**: a quantity used by general agreement of the scientific community

- **experimental value**: a quantitative value measured during an experiment

- **error**: the difference between the accepted value and the experimental value

- **percent error**: the percent that a measured value differs from the accepted value

- **significant figure**: all the digits that can be known precisely in a measurement, plus a last estimated digit

3.1 Using and Expressing Measurements > BIG IDEA

- Scientists express the degree of uncertainty in their measurements and calculations by using significant figures.

- In general, a calculated answer cannot be more precise than the least precise measurement from which it was calculated.
END OF 3.1